COMP314 Midterm Exam 1, Spring 2023

Time allowed: 75 minutes

Total points on exam: 75

Question 1. (20 points) Consider the computational problem OutputContainsInput, abbreviated to Oci, defined as follows. The input to Oci is an ASCII string P. If P is not a SISO Python program, the solution is "no". Otherwise, we may assume P is a SISO Python program and the solutions are defined as follows. A string I is a solution if its length is $\geqslant 3$ and I is a substring of $P(I)$. For example, if $P(\text{"xyz"}) = \text{"abcxyzpqrst"}$, then "xyz" is a solution. In other words, the solution set consists of all input strings I of length 3 or more whose output contains the input I. If there is no string I with this property, the instance is negative and the solution is "no".

Prove that Oci is uncomputable.

Question 2. (20 points) Create a deterministic Turing machine with the following properties. The machine is a transducer, and its alphabet is the ASCII alphabet augmented with the blank symbol. The input is guaranteed to contain only characters from the genetic alphabet $\{\mathtt{C}, \mathtt{A}, \mathtt{G}, \mathtt{T}\}$—so your machine need only work correctly on genetic string inputs. If the input contains exactly one G or at least two T's then the output is the same as the input. Otherwise, the output is "ZZZZ".

Question 3. (20 points) Let L be a language on the genetic alphabet defined as follows:

$$L = \{\mathtt{GGA}^{n+2}\mathtt{T}^{3m}\mathtt{CC} \text{ such that } n > m\},$$

where n and m are nonnegative integers. For example, members of L include "GGAAAAATTTCC" and "GGAAAAAAAATTTTTTCC" but not "GGAAATTTCC".

Prove that L is not regular.

Question 4. (10 points) Consider the computational problem TMprintsGin20, defined as follows. The input is an ASCII description of a Turing machine M. If M prints a G within its first 20 steps on input I, then I is a solution. If there is no such string I, the solution is "no". Is TMprintsGin20 computable? Briefly justify your answer, without necessarily giving a rigorous mathematical proof.

Question 5. (5 points) Suppose someone managed to build a multicore quantum computer—a computer that can simultaneously execute multiple quantum algorithms. Do there exist computational problems which can be solved by the multicore quantum computer but not by a classical computer? Briefly justify your answer.